A Novel Analytical Framework of Fluid Antenna System for 6G Wireless Networks

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***Abstract*—This paper proposes novel closed-form upper-bound expressions of outage probability for fluid antenna systems (FAS) by considering channel correlation among antenna ports. In the FAS, a single antenna port is activated by instantaneously moving the metal fluid on the antenna port with the greatest channel gain. In order to obtain a closed-form outage probability of FAS, we propose a novel analytical framework with Green's matrix approximation to analyze the outage probability of FAS mathematically. To the best of our knowledge, the proposed closed-form expression is the first mathematically tractable and effective theoretical result in the literature.**

***Index Terms*—6G, MIMO, Fluid antenna systems, Matrix approximation, Outage probability, Movable antenna systems.**

I. Introduction

Recently, various mobile devices such as laptops, smart phones, tablets, etc, have emerged with the development of 5G wireless communication systems. To effectively support the data traffic from such mobile devices, various wireless communication technologies are required. One of the most attractive technical solution for 5G mobile communication techniques is the multiple-input multiple-output (MIMO) technology [1]. Basically, MIMO techniques involve multiple antennas at both transmitter and receiver, and they are capable to improve diversity and multiplexing gain of wireless channels. The wireless communication technique with multiple antennas is being expected to provide significant technical impact in future wireless communication systems, i.e., 6G systems. In particular, it will be relatively easy to install many antennas in small physical spaces as the wavelength of radio waves becomes shorter when high frequencies such as THz is used for wireless communication systems. Thus, a massive number of antennas are being considered to used at both transmitter and receiver, and various types of antennas are being studied as technical candidates for cost-effective MIMO solutions.

By the way, inspired by the recent trends on the availability of using liquid metals such as alloys of gallium and tin, the liquid metal antenna has emerged [2]. As a type of liquid metal antenna, the authors of [3] proposed a novel antenna technology called *fluid antenna system (FAS)*, where FAS refers to the antenna that can alter their shape, size, and position based on software-controllable fluids. A fluid antenna is assumed to have many receive antenna ports inside a small linear structure to gain the spatial diversity gain. However, each antenna port inside a small linear structure cannot be assumed to be an independent element and the channel correlation among antenna ports should be considered. In the literature, there exist many studies on mathematical analysis of FAS, but no studies have considered the exact channel correlation among antenna ports.

In this paper, we mathematically model an *exact* channel correlation among antenna ports of FAS, where an arbitrary antenna port has correlation with all antenna ports in the FAS. It is worth noting that existing mathematical analysis frameworks including [3] only assumed that an arbitrary antenna port has spacial correlation with only a certain antenna port. In specific, we propose an analysis method that mathematically approximates the outage probability by approximating the channel correlation matrix with Green's matrix.

The rest of this paper is organized as follows. We describe the proposed system model in Section II, and we mathematically analyze the outage probability with Green's matrix approximation in Section III. In Section IV, we validate our analytical results via computer simulations with numerical examples. Finally, conclusions are drawn in Section V.

II. System Model

In this paper, we follow the same abstraction of FAS as in [3]. We consider the system consisting of a transmitter and a receiver with a single antenna. The receiver is a linear antenna with $W\lambda$ length, and there is a movable fluid inside the antenna. It is assumed that this software-controllable fluid can be placed on N equally distributed preset locations (referred to as antenna port). W is antenna's length parameter and λ means wavelength of this system. The wireless channel vector $\mathbf{h} \in \mathbb{C}^{N\times 1} = \{h_1, h_2, ..., h_N\}^{\mathrm{T}}$ assumes a Rayleigh fading channel model and follows the distribution of $\mathcal{CN}(\mathbf{0}_N, \mathbf{R})$. $\mathbf{R} \in \mathbb{C}^{N\times N}$ is consisted of channel correlation coefficients. Then, the received signal is given by,

$$\mathbf{y} = \sigma_h \mathbf{h}\mathbf{x} + \mathbf{n}. \tag{1}$$

σ_h means channel power. It is assumed that the transmitted signal and the additive white Gaussian noise (AWGN) are expressed as $\mathbf{x} \in \mathbb{C}^{N\times 1}$ and $\mathbf{n} \in \mathbb{C}^{N\times 1}$, respectively, and $\mathbf{n}$ follows the distribution of $\mathcal{CN}(\mathbf{0}, \mathbf{I}_N \times N_0)$.

A. Previous FAS Channel model

In [3], taking the first port as a reference port, the distance between k-th port and the first port is following:

$$\Delta d_k = \frac{k-1}{N-1} W\lambda, \text{ for } k = 2, ..., N. \tag{2}$$

In this model, channel correlation coefficients μ_k following Jakes' model is given by,

$$\mu_1 = 0, \quad \mu_k = \sigma_h^2 J_0\left(2\pi \frac{\Delta d_k}{\lambda}\right) = \sigma_h^2 J_0\left(\frac{k-1}{N-1} W\right), \quad \text{for } k = 2, ..., N, \tag{3}$$

where $J_0(.)$ is the zero-order Bessel function of the first kind. The k-th correlated FAS channel model is given by,

$$\begin{cases} h_1 = \sigma_h a_1 + j\sigma_h b_1, \\ h_k = \sigma_h\left(\sqrt{1-\mu_k^2} a_k + \mu_k a_1\right) + j\sigma_h\left(\sqrt{1-\mu_k^2} b_k + \mu_k b_1\right), \\ \qquad \text{for } 2 \le k \le N. \end{cases} \tag{4}$$

j is a complex number unit, a_k and b_k follows identically and independently distributed (i.i.d.) Gaussian distribution with $\mathcal{N}(0, \frac{1}{2})$. By [3], FAS can quickly switch to the antenna port with the maximum magnitude of channel, so the receiving channel for FAS is following:

$$h_{\text{FAS}} = \max\{|h_1|, |h_2|, ..., |h_N|\}. \tag{5}$$

B. Proposed FAS Channel model

We propose correlated Rayleigh fading channel model between arbitrary k-th port and l-th port following Jakes' model. The distance between k-th port and l-th port is following:

$$\Delta d_{k,l} = \frac{k-l}{N-1} W\lambda, \text{ for } k = 1, 2, ..., N. \tag{6}$$

In this model, channel correlation coefficients in $\mathbf{R}$ following Jakes' model is given by,

$$R_{k,l} = \sigma_h^2 J_0\left(2\pi \frac{\Delta d_{k,l}}{\lambda}\right) = \sigma_h^2 J_0\left(\frac{k-l}{N-1} W\right), \quad \text{for } k = 1, 2, ..., N. \tag{7}$$

Since $\mathbf{R} = \mathbb{E}\left\{\mathbf{h}\mathbf{h}^{\mathbf{H}}\right\}$, correlation matrix has to be multiplied to channel is $\mathbf{R}^{\frac{1}{2}}$. So we have to eigenvalue decomposition to calculate $\mathbf{R}^{\frac{1}{2}}$. After the eigenvalue decomposition of $\mathbf{R}$, $\mathbf{R}$ is expressed as $\mathbf{R} = \mathbf{U}\mathbf{D}\mathbf{U}^H$. $\mathbf{D} \in \mathbb{R}^{N\times N}$ is a diagonal matrix with eigenvalues of $\mathbf{R}$, and $\mathbf{U} \in \mathbb{R}^{N\times N}$ is a matrix with eigenvectors related to eigenvalues. When defined as $\mathbf{S} = \mathbf{U}\mathbf{D}^{\frac{1}{2}}$, $\mathbf{S}$ becomes a matrix composed of $S_{k,l}\,(k, l \in \{1, ..., N\})$ which is the correlation coefficient between channels.

Considering correlation with all antenna ports, the k-th correlated FAS channel model is given by,

$$h_k = \sum_{l=1}^{N} S_{k,l}(a_l + jb_l), \tag{8}$$

a_l and b_l follows identically and independently distributed (i.i.d.) Gaussian distribution with $\mathcal{N}(0, \frac{1}{2})$. Maximum channel magnitude is expressed as following:

$$M = \arg_k \max\{|h_1|, |h_2|, ..., |h_N|\}. \tag{9}$$

Therefore, outage events at the receiver is following:

$$\left\{\log_2\left(1 + \sqrt{h_M^2 \gamma_M}\right) < R\right\} = \left\{h_M < \sqrt{\gamma_{th}/\gamma_M}\right\}. \tag{10}$$

γ_M is an average transmit SNR on the active antenna port, R indicates the required target data rate and $\gamma_{th} \triangleq 2^R - 1$.

$\mathbf{W}$ means $\mathbf{C}^{-1}$, $|.|$ means matrix, and $\mathbf{C}^{-1}$ with elements $p_{i,j}$ is $1 \le i, j \le N$. $\Gamma(.)$ is a Gamma function, $\gamma(.)$ is an incomplete Gamma function. Using the equation of the CDF, the equation of outage probability of this system is given by,

$$P_{\text{out}}(\gamma_{th}) \approx F_{|g_1|,|g_2|,...,|g_N|}(\sqrt{\gamma_{th}/\gamma_M}, \cdots, \sqrt{\gamma_{th}/\gamma_M}) \tag{11}$$

III. Outage Probability Analysis

In this section, we find closed-form distribution of correlated channel and mathematically analyze the outage probability of the FAS. In the existing literature, the correlation between antenna ports in literature [3] only considered the correlation between the arbitrary antenna port and the first port. In addition, the channel distribution have been analyzed as shown in formula (11) using conditional probabilities under the assumption that the first port is the reference port and all channels on the other ports are independent.

$$F_{|h_1|,|h_2|,...,|h_N|}(r_1, r_2, ..., r_N) = F_{|h_2|,|h_3|,...,|h_N|}(r_2, r_3, ..., r_N)\, F_{|h_1|}(r_1), \tag{12}$$

However, the above analysis method is not correct because it did not take into account the correlation between all antennas. In addition, the final expression of the outage probability derived from this method is not given as a closed form of expression.

Therefore, the distribution of channels should be analyzed considering correlation for all antenna ports. In literature [4], the channel distribution has been analyzed when an arbitrary correlation coefficient between channels are given. However, the final expression of the cumulative distribution function (CDF) derived in [4] is very complex, making it difficult to obtain meaningful insights.

So in this paper, channels with correlations have been analyzed using the Green's matrix approximation in the literature [5]. The Green matrix means a new matrix $\mathbf{C}$ created by approximating the value closest to the existing correlation matrix $\mathbf{R}$ with the condition of $\mathbf{C}^{-1}$ to be a tridiagonal matrix. A symmetric, irreducible nonsingular matrix, is tridiagonal and a tridiagonal matrix refers to a matrix in which the elements are zero except for the diagonal of the matrix and the upper diagonal located directly above and below the diagonal.

$$\mathbf{C} = \begin{pmatrix} u_1v_1 & u_1v_2 & \cdots & u_1v_n \\ u_1v_2 & u_1v_2 & \cdots & u_2v_n \\ \vdots & \vdots & \ddots & \vdots \\ u_nv_2 & u_nv_2 & \cdots & u_nv_n \end{pmatrix}, \tag{13}$$

$$F_{|g_1|,|g_2|,\ldots,|g_N|}(R_1,R_2,\ldots,R_N) = |\mathbf{W}| \sum_{i_1=0}^{\infty}\sum_{i_2=0}^{\infty}\cdots\sum_{i_{N-1}=0}^{\infty} \frac{|p_{1,2}|^{2i_1}|p_{2,3}|^{2i_2}\cdots|p_{N-1,N}|^{2i_{N-1}}}{p_{1,1}^{i_1+1}p_{2,2}^{i_1+i_2+1}\cdots p_{N-1,N-1}^{i_{N-2}+i_{N-1}+1}\;p_{N,N}^{i_{N-1}+1}} \prod_{j=1}^{N-1}(i_j!\Gamma(i_j+1))^{-1}$$
$$\times\gamma(i_1+1,\frac{1}{2}p_{1,1}R_1^2)\gamma(i_1+i_2+1,\frac{1}{2}p_{2,2}R_2^2)\cdots\gamma(i_{N-2}+i_{N-1}+1,\frac{1}{2}p_{N-1,N-1}R_{N-1}^2)\gamma(i_{N-1}+1,\frac{1}{2}p_{N,N}R_N^2) \quad (15)$$

with u_i and $v_i, i = 1, 2, ..., N$. Because the diagonal elements of correlation matrix is 1, $u_i/v_i = 1$ and $u_i = 1/v_i$. So Green's matrix satisfying the above condition is calculated by the nonlinear equation below.

$$\begin{array}{llll} v_2 = R_{2,1}v_1 & & & \\ v_3 = R_{2,1}v_1 & v_3 = R_{3,2}v_2 & & \\ & v_4 = R_{4,2}v_2 & v_4 = R_{4,3}v_3 & \\ \vdots & \vdots & v_5 = R_{5,3}v_3 & \ddots\ddots \\ v_n = R_{n,1}v_1 & v_n = R_{n,2}v_2 & \cdots\cdots & v_n = R_{n,n-1}v_{n-1} \end{array} \quad (14)$$

This nonlinear equation can be solved by Levenberg-Marquardt algorithm. If there is a Green matrix satisfied above all conditions, when the channel correlation existed, the the joint CDF of correlated channel can be mathematically calculated by closed-form equation (15).

IV. Simulation Results

In this section, we validate the outage probability performance and CDF of channel gain in the fluid antenna system considering channel correlation between evenly spaced antenna ports through Monte-Carlo simulations and we verify analytical results by comparing them with computer simulations. We set the fluid antenna length to 2λ, Fig. 1 shows the CDF of the channel gain for varying the number of antenna ports. As R increases, the accuracy of the mathematically analyzed CDF tends to increase.

Fig. 2 shows the outage probability performance of the fluid antenna system for varying transmit SNR. It can be seen that as the number of antenna ports increases, channel selection diversity improves, resulting in a decrease in outage probability. In addition, it is worth noting that our mathematical analysis using Green's matrix approximation represents the upper bound of the simulation results.

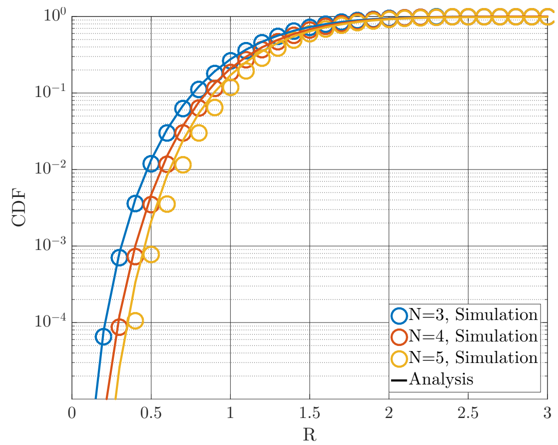

Fig. 1. CDF of channel gain in FAS for varying R

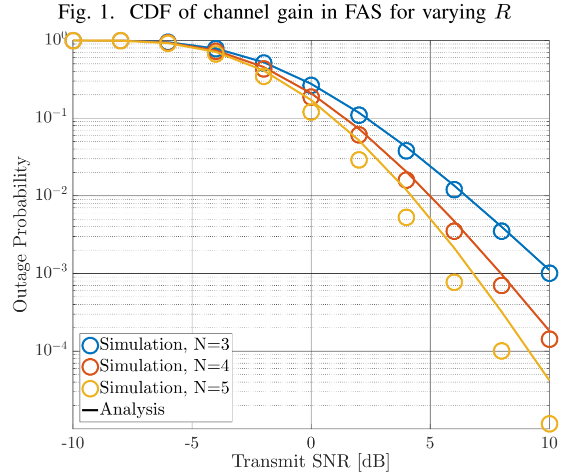

Fig. 2. Outage probability for varying transmit SNR

V. Conclusions

In this paper, we proposed a novel and exact channel model considering correlations among antenna ports for the fulid antenna system (FAS), and mathematically analyzed its outage probability for 6G wireless networks. In particular, we proposed a novel analytical framework to mathematically analyze the outage probability of the FAS by using Green's matrix approximation. The proposed analytical results were validated through computer simulation results.

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